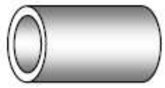
3–70 A50-m-long section of a steam pipe whose outer diameter is 10 cm passes through an open space at 15°C. The average temperature of the outer surface of the pipe is measured to be 150°C. If the combined heat transfer coefficient on the outer surface of the pipe is 20 $W/m^2 \cdot °C$, determine (*a*) the rate of heat loss from the steam pipe, (*b*) the annual cost of this energy lost if steam is generated in a natural furnace that has an efficiency of 75 percent and the price of natural gas is $0.52/therm (1 therm _ 105,500 kJ), and (*c*) the thickness of fiberglass insulation (*k* = 0.035 W/m · °C) needed in order to save 90 percent of the heat lost. Assume the pipe temperature to remain constant at 150°C.

Solution:

Analysis (*a*) The rate of heat loss from the steam pipe is



$$A_o = \pi DL = \pi(0.1\,\text{m})(50\,\text{m}) = 15.71\,\text{m}^2$$

$$\dot{Q}_{bare} = h_o A(T_s - T_{air}) = (20\,\text{W/m}^2.°\text{C})(15.71\,\text{m}^2)(150-15)°\text{C} = \mathbf{42{,}412\,W}$$

(*b*) The amount of heat loss per year is

$$Q = \dot{Q}\Delta t = (42.412\,\text{kJ/s})(365\times 24\times 3600\,\text{s/yr}) = 1.337\times 10^9\,\text{kJ/yr}$$

The amount of gas consumption from the natural gas furnace that has an efficiency of 75% is

$$Q_{gas} = \frac{1.337\times 10^9\,\text{kJ/yr}}{0.75}\left(\frac{1\,\text{therm}}{105{,}500\,\text{kJ}}\right) = 16{,}903\,\text{therms/yr}$$

The annual cost of this energy lost is

Energy cost = (Energy used)(Unit cost of energy)

= (16,903 therms/yr)($0.52 / therm) = **$8790/yr**

(*c*) In order to save 90% of the heat loss and thus to reduce it to 0.1×42,412 = 4241 W, the thickness of insulation needed is determined from

$$\dot{Q}_{insulated} = \frac{T_s - T_{air}}{R_o + R_{insulation}} = \frac{T_s - T_{air}}{\dfrac{1}{h_o A_o} + \dfrac{\ln(r_2/r_1)}{2\pi kL}}$$

Substituting and solving for r_2, we get

$$4241\,\text{W} = \frac{(150-15)°\text{C}}{\dfrac{1}{(20\,\text{W/m}^2.°\text{C})[(2\pi r_2(50\,\text{m})]} + \dfrac{\ln(r_2/0.05)}{2\pi(0.035\,\text{W/m}.°\text{C})(50\,\text{m})}} \longrightarrow r_2 = 0.0692\,\text{m}$$

Then the thickness of insulation becomes

$$t_{insulation} = r_2 - r_1 = 6.92 - 5 = \mathbf{1.92\,cm}$$

A spherical tank, 1 m in diameter, is maintained at a temperature of 120◦C and exposed to a convection environment. With h=25W/m2 · ◦C and T_∞ =15◦C, what thickness of urethane foam should be added to ensure that the outer temperature of the insulation does not exceed 40◦C? What percentage reduction in heat loss results from installing this insulation?

Solution:

$$q\text{ (no ins.)} = hA(T_w - T_\infty) = (25)(4\pi)(0.5)^2(120-15) = 8247\text{ W}$$

$$k_{\text{foam}} = \frac{18\text{ mW}}{\text{m}\cdot{}^\circ\text{C}}$$

$$q = \frac{4\pi k(T_i - T_0)}{\frac{1}{r_i} - \frac{1}{r_0}} = h4\pi r_0^2(T_0 - T_\infty)$$

$$\frac{(0.018)(120-40)}{\frac{1}{0.5} - \frac{1}{r_0}} = (25)r_0^2(40-15)$$

$$r_0 = 0.5023\text{ m}$$

$$\text{thk} = r_0 - r_i = 0.023\text{ m}$$

$$q\text{ (w/ ins)} = (25)(4\pi)(0.5023)^2(40-15) = 1982\text{ W}$$

Tutorial heat generation

Q1

A 2-kW resistance heater wire whose thermal conductivity is k =15 W/m · °C has a diameter of D= 4 mm and a length of L =0.5 m, and is used to boil water (Figure below). If the outer surface temperature of the resistance wire is T_s =105°C, determine the temperature at the center of the wire.

Solution:

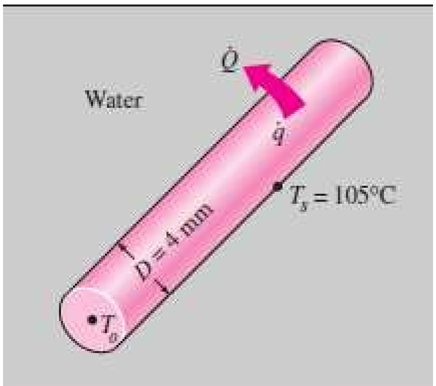
Assumptions

1. Heat transfer is steady since there is no change with time.

2. Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no change in the axial direction.

3. Thermal conductivity is constant.

4. Heat generation in the heater is uniform.



$$\dot{q} = \frac{q}{V} = \frac{q}{\pi r_o^2 L} = \frac{2000}{\pi(0.002)^2(0.5)} = 0.318 * 10^9\ W/m^3$$

$$T_{max} - T_s = \frac{\dot{q} r_o^2}{4k}$$

The temperature at the center of the wire is the maximum temperature

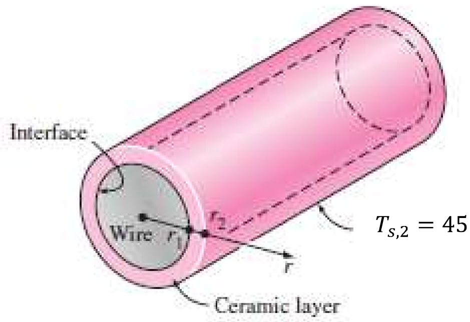
$$T_{max} = T_s + \frac{\dot{q} r_o^2}{4k} = 105 + \frac{(0.318*10^9)(0.002)^2}{4(15)} = 126\ ^\circ\text{C}.$$

Q2

Consider a long resistance wire of radius r_1 = 0.2 cm and thermal conductivity k_{wire} = 15 W/m · °C in which heat is generated uniformly as a result of resistance heating at a constant rate of $\dot{q}$ = 50 W/cm^3 (see Figure below). The wire is embedded in a 0.5-cm-thick layer of ceramic whose thermal conductivity is $k_{ceramic}$ 1.2 = W/m · °C. If the outer surface temperature of the ceramic layer is measured to be T_s = 45°C, determine the temperatures at the center of the resistance wire and the interface of the wire and the ceramic layer under steady conditions.

Solution:

The temperature distribution in the holow cylinder without heat generation is expressed as below:



$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln({}^{r_1}/_{r_2})} \ln\left(\frac{r}{r_2}\right) + T_{s,2} \quad ....a$$

The temperature distribution in the solid wire

with heat generation is expressed as below:

$$T(r) - T_s = \frac{\dot{q} r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right) \quad .... b$$

$$-k_{wire} A \frac{dT_{wire}(r_1)}{dr} = -k_{ceramic} A \frac{dT_{ceramic}(r_1)}{dr} \quad ......c$$

From equation a:

$$\frac{dT}{dr} = \frac{T_{s,1} - T_{s,2}}{r\ln({}^{r_1}/_{r_2})}$$

$$\frac{dT_{ceramic}(r_1)}{dr} = \frac{T_{s,1} - T_{s,2}}{r_1 \ln({}^{r_1}/_{r_2})} \quad ....d$$

From equation b

$$\frac{dT}{dr} = -\frac{\dot{q}}{2k} r$$

$$\frac{dT_{wire}(r_1)}{dr} = -\frac{\dot{q}}{2k_{wire}} r_1 \quad ....e$$

Substitute equation (d) and (e) in (c) gives:

$$k_{wire} \frac{\dot{q}}{2k_{wire}} r_1 = -k_{ceramic} \frac{T_{s,1} - T_{s,2}}{r_1 \ln({}^{r_1}/_{r_2})}$$

$T_{s,1} = T_{s,2} + \frac{\dot{q}r_1^2}{2k_{ceramic}} ln\frac{r_2}{r_1} = 45 + \frac{50*10^6(0.002)^2}{2*1.2} \ln(\frac{0.007}{0.002})$

$T_{s,1} = 149.4$ °C

At $r = 0$, T=T_{max}

$T_{max} = T_{s,1} + \frac{\dot{q}r_1^2}{4k_{wire}} = 149.4 + \frac{50*10^6(0.002)^2}{4(15)} = 152.7$ °C

2.4 Thermal Contact Resistance

Imagine two solid bars brought into contact as indicated in Figure 2-19, with the sides of the bars insulated so that heat flows only in the axial direction. The materials may have different thermal conductivities, but if the sides are insulated, the heat flux must be the same through both materials under steady-state conditions. Experience shows that the actual temperature profile through the two materials varies approximately as shown in Figure 2-19*b*. The temperature drop at plane 2, the contact plane between the two materials, is said to be the result of a *thermal contact resistance*. Performing an energy balance on the two materials, we obtain

$$q = k_A A \frac{T_1 - T_{2A}}{\Delta x_A} = \frac{T_{2A} - T_{2B}}{1/h_c A} = k_B A \frac{T_{2B} - T_3}{\Delta x_B}$$

or

$$q = \frac{T_1 - T_3}{\Delta x_A/k_A A + 1/h_c A + \Delta x_B/k_B A}$$

where

$1/h_cA$ is called the thermal contact resistance and

h_c is called the contact coefficient.

The physical mechanism of contact resistance may be better understood by examining a joint in more detail, as shown in Figure 2-20. No real surface is perfectly smooth, and the actual surface roughness is believed to play a central role in determining the contact resistance. There are two principal contributions to the heat transfer at the joint:

1. The solid-to-solid conduction at the spots of contact

2. The conduction through entrapped gases in the void spaces created by the contact

The second factor is believed to represent the major resistance to heat flow, because the thermal conductivity of the gas is quite small in comparison to that of the solids.

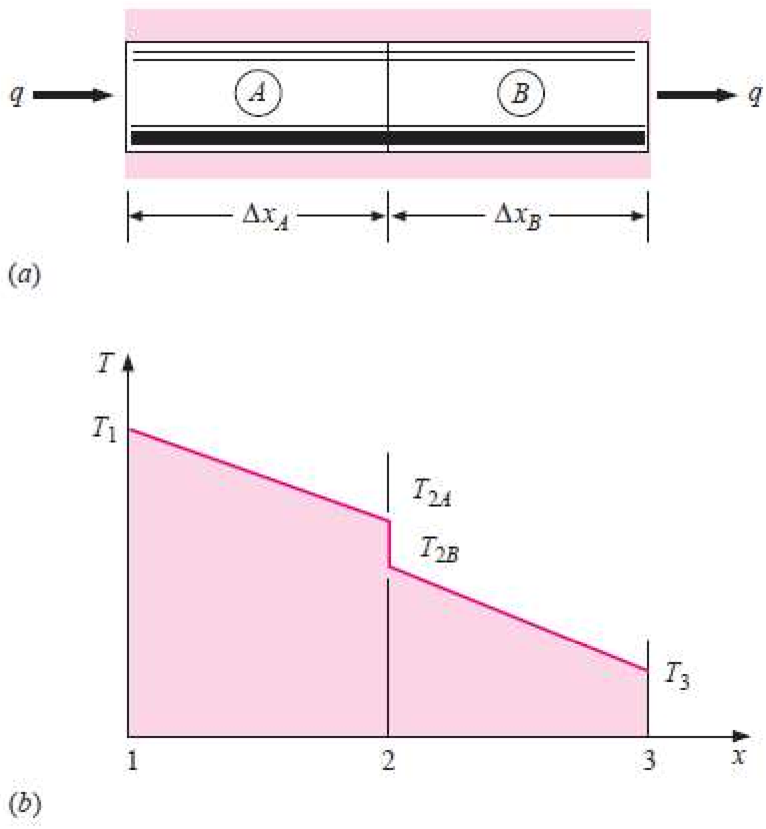


Figure 2-19 Illustrations of thermal-contact-resistance effect: (*a*) physical situation; (*b*) temperature profile.

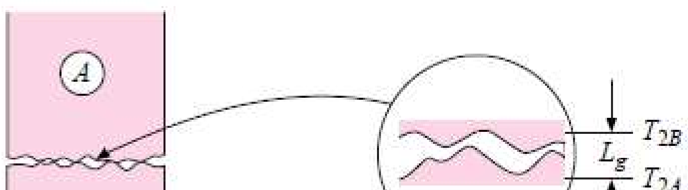


Figure 2-20 Joint-roughness model for analysis of thermal contact resistance.

Table 2.1 Contact conductance of typical surfaces.

Surface type	Roughness, μ in	Roughness, μm	Temperature, °C	Pressure, atm	$1/h_c$, h · ft² · °F/Btu	$1/h_c$, m² · °C/W × 10⁴
416 Stainless, ground, air	100	2.54	90–200	3–25	0.0015	2.64
304 Stainless, ground, air	45	1.14	20	40–70	0.003	5.28
416 Stainless, ground, with 0.001-in brass shim, air	100	2.54	30–200	7	0.002	3.52
Aluminum, ground, air	100	2.54	150	12–25	0.0005	0.88
	10	0.25	150	12–25	0.0001	0.18
Aluminum, ground, with 0.001-in brass shim, air	100	2.54	150	12–200	0.0007	1.23
Copper, ground, air	50	1.27	20	12–200	0.00004	0.07
Copper, milled, air	150	3.81	20	10–50	0.0001	0.18
Copper, milled, vacuum	10	0.25	30	7–70	0.0005	0.88

Two 3.0-cm-diameter 304 stainless-steel bars, 10 cm long, have ground surfaces and are exposed to air with a surface roughness of about 1 μm. If the surfaces are pressed together with a pressure of 50 atm and the two-bar combination is exposed to an overall temperature difference of 100°C, calculate the axial heat flow and temperature drop across the contact surface.

■ **Solution**

The overall heat flow is subject to three thermal resistances, one conduction resistance for each bar, and the contact resistance. For the bars

$$R_{th} = \frac{\Delta x}{kA} = \frac{(0.1)(4)}{(16.3)\pi(3 \times 10^{-2})^2} = 8.679°C/W$$

From table 2.1 $\frac{1}{h_c}$=5.28*10^{-4}

$$R_c = \frac{1}{h_c A} = \frac{(5.28 \times 10^{-4})(4)}{\pi(3 \times 10^{-2})^2} = 0.747°\text{C/W}$$

The total thermal resistance is therefore

$$\sum R_{th} = (2)(8.679) + 0.747 = 18.105$$

and the overall heat flow is

$$q = \frac{\Delta T}{\sum R_{th}} = \frac{100}{18.105} = 5.52\ \text{W} \quad [18.83\ \text{Btu/h}]$$

The temperature drop across the contact is found by taking the ratio of the contact resistance to the total thermal resistance:

$$\Delta T_c = \frac{R_c}{\sum R_{th}} \Delta T = \frac{(0.747)(100)}{18.105} = 4.13°\text{C} \quad [39.43°\text{F}]$$

In this problem the contact resistance represents about 4 percent of the total resistance.

2.5 Heat Transfer from Extended Surfaces

There are many different situations that involve such combined conduction–convection effects, the most frequent application is one in which an extended surface is used specifically to enhance heat transfer between a solid and an adjoining fluid. Such an extended surface is termed a fin.

Consider the plane wall of Figure 2.11a. If T_s is fixed, there are two ways in which the heat transfer rate may be increased. The convection coefficient h could be increased by increasing the fluid velocity, and/or the fluid temperature T could be reduced. However, there are many situations for which increasing h to the maximum possible value is either insufficient to obtain the desired heat transfer rate or the associated costs are prohibitive. Such costs are related to the blower or pump power requirements needed to increase h through increased fluid motion. Moreover, the second option of reducing T is often impractical. Examining Figure 2.11b, however, we see that there exists a third option. That is, the heat transfer rate may be increased by increasing the surface area across which the convection occurs. This may be done by employing fins that extend from the wall into the surrounding fluid.

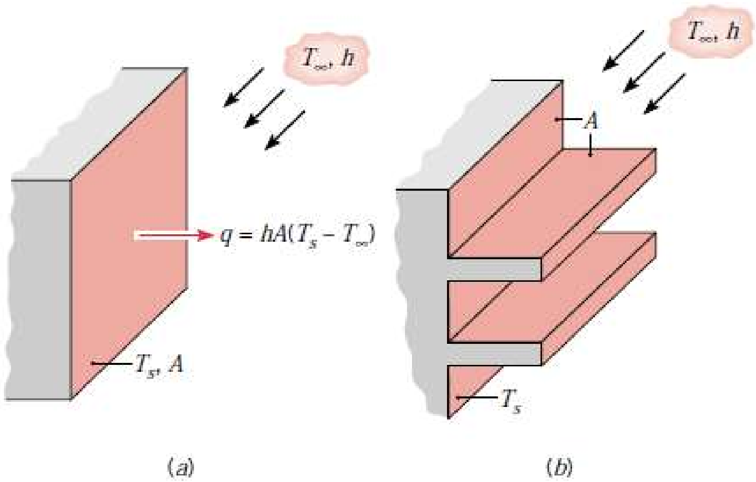


FIGURE 2.121 Use of fins to enhance heat transfer from a plane wall. (a) Bare surface. (b) Finned surface.

The thermal conductivity is the effective parameter in the design of fins. High thermal conductivity gives high heat transfer rate from the surface of the extended surfaces (fins)

Examples of fin applications are easy to find. Consider the arrangement for cooling engine heads on motorcycles and lawn mowers or for cooling electric power transformers. Consider also the tubes with attached fins used to promote heat exchange between air and the working fluid of an air conditioner. Two common finned tube arrangements are shown in Figure 2.12.

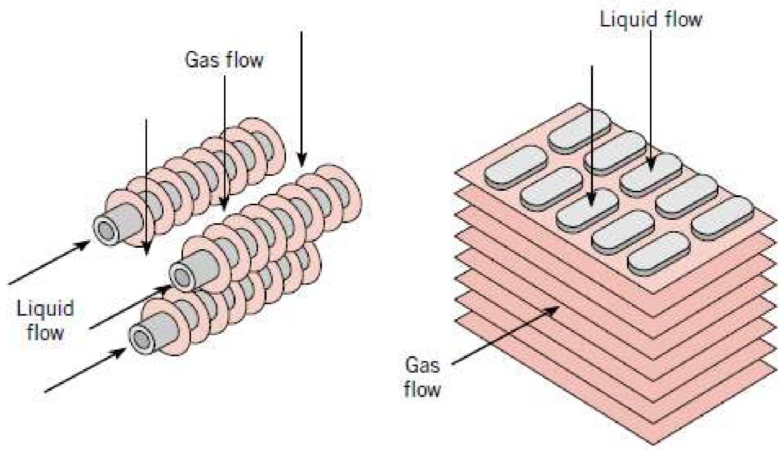


FIGURE 2.12 Schematic of typical finned-tube heat exchangers.

Different fin configurations are illustrated in Figure 2.13. A straight fin is any extended surface that is attached to a plane wall. It may be of uniform cross-sectional area, or its cross-sectional area may vary with the distance x from the wall.

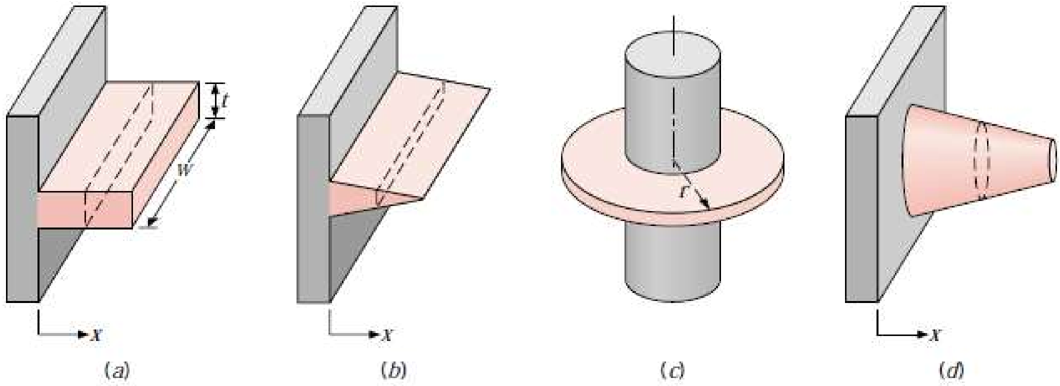


FIGURE 2.13 Fin configurations. (a) Straight fin of uniform cross section. (b) Straight fin of non-uniform cross section. (c) Annular fin. (d) Pin fin.

2.5.1 A General Conduction Analysis

Assumption:

1. No heat generation
2. Study state
3. Constant thermal conductivity.
4. Neglect radiation.
5. The convection heat transfer coefficient *h* is uniform over the surface.

Figure 2.14 shows the energy balance on the element on the extended surface.

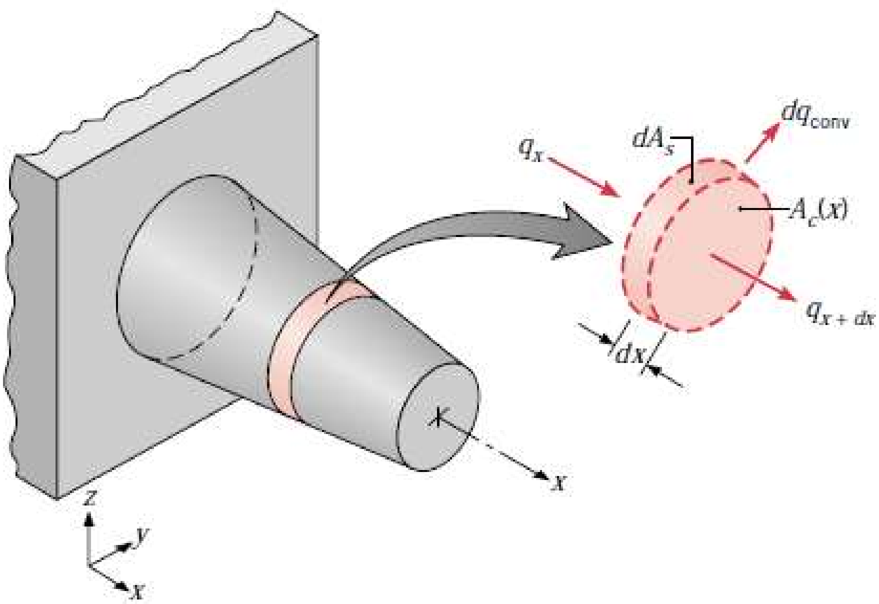


FIGURE 2.14 Energy balance for an extended surface.

$$q_x = q_{x+dx} + dq_{conv}$$

From Fourier's law we know that

$$q_x = -kA_c \frac{dT}{dx}$$

where A_c is the cross-sectional area, which may vary with x. Since the conduction heat rate at $x + dx$ may be expressed as

$$q_{x+dx} = -kA_c \frac{dT}{dx} - k \frac{d}{dx}\left(A_c \frac{dT}{dx}\right) dx$$

The convection heat transfer rate may be expressed as

$$dq_{conv} = hdA_s(T - T_\infty)$$

where dA_s is the surface area of the differential element. Substituting the foregoing rate equations into the energy balance

$$-kA_c \frac{dT}{dx} = -kA_c \frac{dT}{dx} - k \frac{d}{dx}\left(A_c \frac{dT}{dx}\right) dx + hdA_s(T - T_\infty)$$

$$\frac{d}{dx}\left(A_c \frac{dT}{dx}\right) - \frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0$$

$$A_c \frac{d^2T}{dx^2} + \frac{dA_c}{dx}\frac{dT}{dx} - \frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0$$

Or

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{h}{kA_c}\frac{dA_s}{dx}\right)(T - T_\infty) = 0 \quad \text{(general equation)}$$

This result provides a general form of the energy equation for an extended surface.

2.5.2 Fins of Uniform Cross-Sectional Area

We begin with the simplest case of straight rectangular and pin fins of uniform cross section (Figure 2.15). Each fin is attached to a base surface of temperature $T(x=0) = T_b$ and extends into a fluid of temperature T_∞. For the prescribed fins, A_c is a constant and $A_s = Px$, where A_s is the surface area measured from the base to x and P is the fin perimeter. Accordingly, with

$dA_c / dx = 0$ (uniform)

and

$dA_s / dx = P$,

apply the condition in the general heat equation